End Sem(III) —
Math (GE – 3)

2021

Time : 3 hours

Full Marks : 100

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the Groups as directed.

Group – A

(Compulsory)

1. Answer the following questions : $1 \times 10 = 10$

   (a) Define monotonic sequence.

   (b) Define greatest lower bound of a sequence.

   (c) Define convergent sequence.

   (d) Define D′Alembert's ratio test.

   (e) Define Group.

   $\frac{x_n}{n}$

   (f) Define Abelian Group.

   (g) Define Cyclic Group.

(h) Solve $\frac{d^2y}{dx^2} - 4y = 0$.

(i) Write the general solution of $y = px + \frac{a}{p}$.

(j) Find the C. F. of $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

2. Prove that the identity element of a group is unique. 5

3. Solve $(D^2 + 9)y = \cos 3x$. 5

Group – B

Answer any **four** questions of the following :

4. (a) Prove that every convergent sequence is bounded. 10

(b) State and prove Cauchy's general principle of convergence. 10

5. (a) Prove that the infinite series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p}$ ...... to ∞ is convergent if $p > 1$ and divergent $p \leq 1$. 10

(b) State the prove Pringsheim's Theorem. 10

6. (a) Prove that $(ab)^{-1} = b^{-1}a^{-1}$, a, b be any elements of a Group. 10

(b) Prove that the set of real numbers forms an Abelian Group under addition. 10

7. State and prove Lagrange's Theorem. 20

8. (a) Solve the differential equation $y = 2px + y^3p^3$. 10

(b) Find the orthogonal trajectory of the curve $r = a(1 + \cos\theta)$. 10

9. (a) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x$. 10

(b) Solve $\frac{dx}{y+z} = \frac{dy}{z+x} = \frac{dz}{x+y}$. 10

c.f

$\frac{dy}{dx} = -\frac{dx}{dy}$

$\frac{dr}{d\theta} = -\frac{d\theta}{dr}$